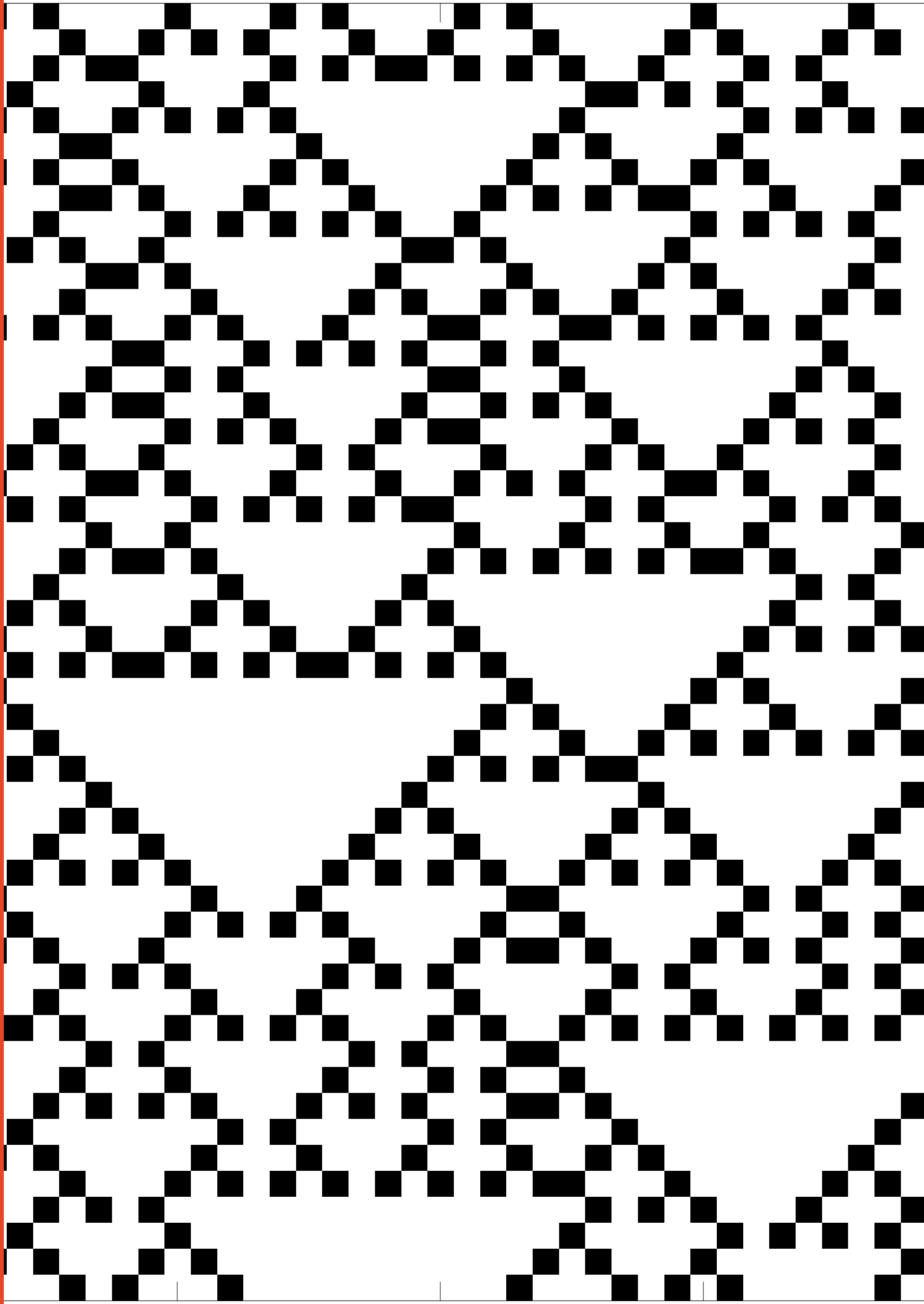
Information dynamics – Part II – Information storage

Dr. Joseph Lizier

THE UNIVERSITY OF SYDNEY



Information dynamics Part II: session outcomes

- Understand measures for **information storage**
- Apply JIDT using AutoAnalyser and extensions of code it produces to analyse information storage in complex systems data sets.

- Primary references (for Info Dynamics II and III sessions):
  - J.T. Lizier, "JIDT: An information-theoretic toolkit for studying the dynamics of complex systems", Frontiers in Robotics and AI, 1:11, 2014; appendix A.2 and A.3
  - J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013; chapter 3, 4
  - Bossomaier, Barnett, Harré, Lizier, "An Introduction to Transfer Entropy: Information Flow in Complex Systems", Springer, Cham, 2016; chapter 4 (sections 4.1-4.3); section 5.1

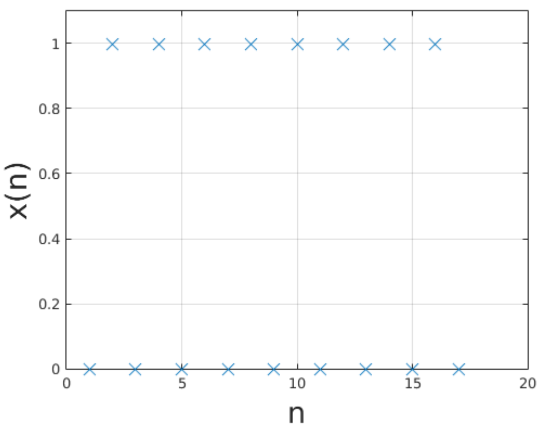
Information storage

- How much information from the past of the variable helps us predict its next state?

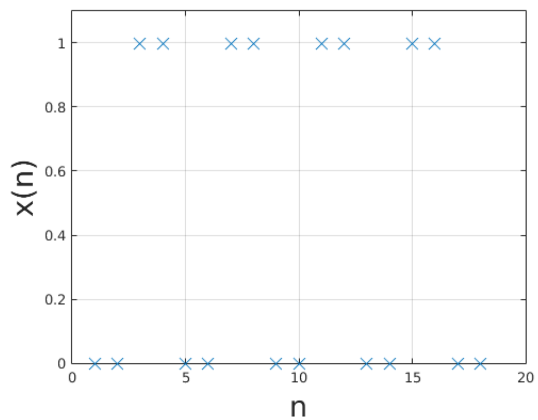
- Or, in modelling the dynamics of the variable, how much information storage would we include in that model by accounting for the past influence of that variable (as a first step)?

Information storage – using our intuition

- In each example:
  - Try to predict the next value of the variable (beyond what is shown).
  - What assumptions did you make?
  - Where specifically did you take the information from to make that prediction?
  - How much information did you get from the past?



(a)



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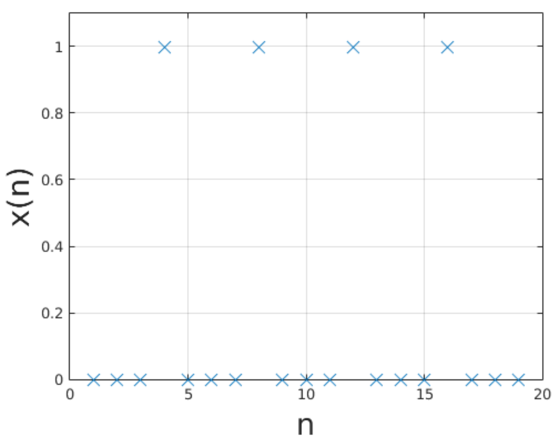

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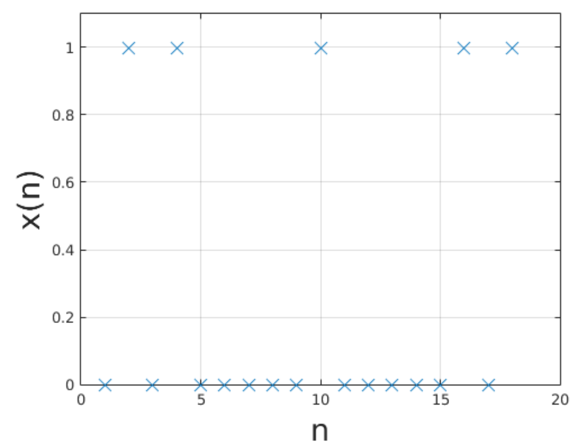
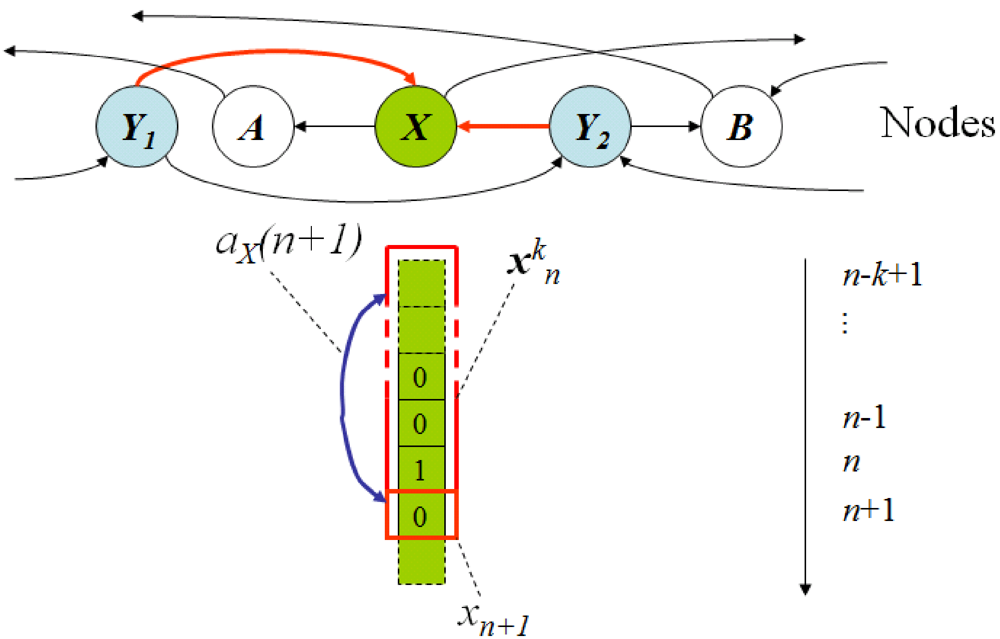


(d)

Active information storage

– How much information about the next observation X_n of process X can be found in its past state $\boldsymbol{X}_n^{(k)} = \{X_{n-k+1}, \ldots, X_{n-1}, X_n\}$?



Active information storage

$$A_X = \lim_{k\to\infty} I(\boldsymbol{X}_n^{(k)}; X_{n+1})$$

$$A_X(k) = I(\boldsymbol{X}_n^{(k)}; X_{n+1})$$

$$A_X = H(X_{n+1}) - H_{\mu X}$$

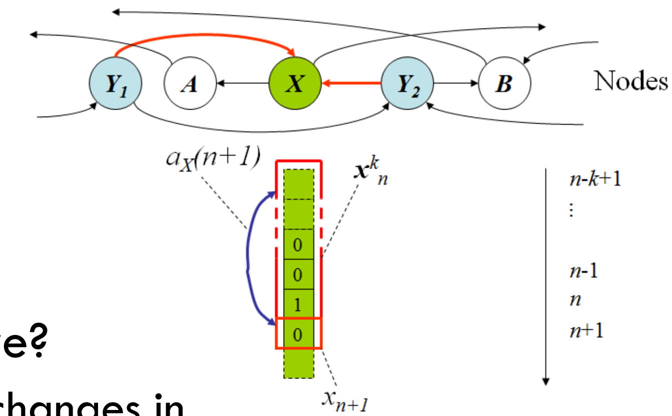
$$A_X(k) = \left\langle \log_2 \frac{p(x_{n+1}|\boldsymbol{x}_n^{(k)})}{x_{n+1}} \right\rangle$$

$$a_X(k) = \log_2 \frac{p(x_{n+1}|\boldsymbol{x}_n^{(k)})}{x_{n+1}}$$

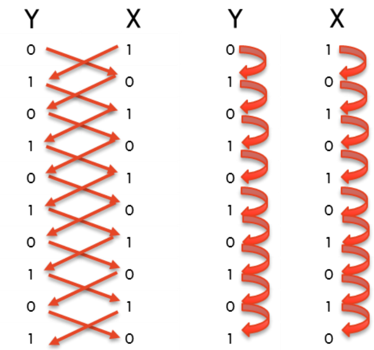
– AIS: Average information from past state that is in use in predicting the next value
– Local AIS: information from a specific past state in use in predicting specific next value

J.T. Lizier, M. Prokopenko, A.Y. Zomaya, "*Detecting Non-trivial Computation in Complex Dynamics*", Proc. ECAL, pp. 895-904 (2007).
J.T. Lizier, M. Prokopenko, & A.Y. Zomaya, "*Local measures of information storage in complex distributed computation*", Information Sciences 208, 39 (2012)
J.T. Lizier, "*JIDT: An information-theoretic toolkit for studying the dynamics of complex systems*", Frontiers in Robotics and AI, 1:11, 2014; appendix A.2 and A.3

Active information storage: interpretations



- Captures total memory and nonlinear effects
  - Autocorrelation is just linear component from each past value separately.
- What types of information storage does A_X capture?
  - Active storage in dynamics (as opposed to passive changes in underlying structure);
  - Internal (causally) stored information;
  - Distributed information storage via feedback and feedforward loops, i.e. recurrent connections (network effects);
  - Input-driven storage: Patterns in input dynamics driving a variable
- All of these are intrinsically *modelled* as information storage to an observer when we account for the information here.



M. Wibral, J. T. Lizier, S. Vögler, V. Priesemann, and R. Galuske, "*Local active information storage as a tool to understand distributed neural information processing*", Frontiers in Neuroinformatics 8, 1+ (2014).

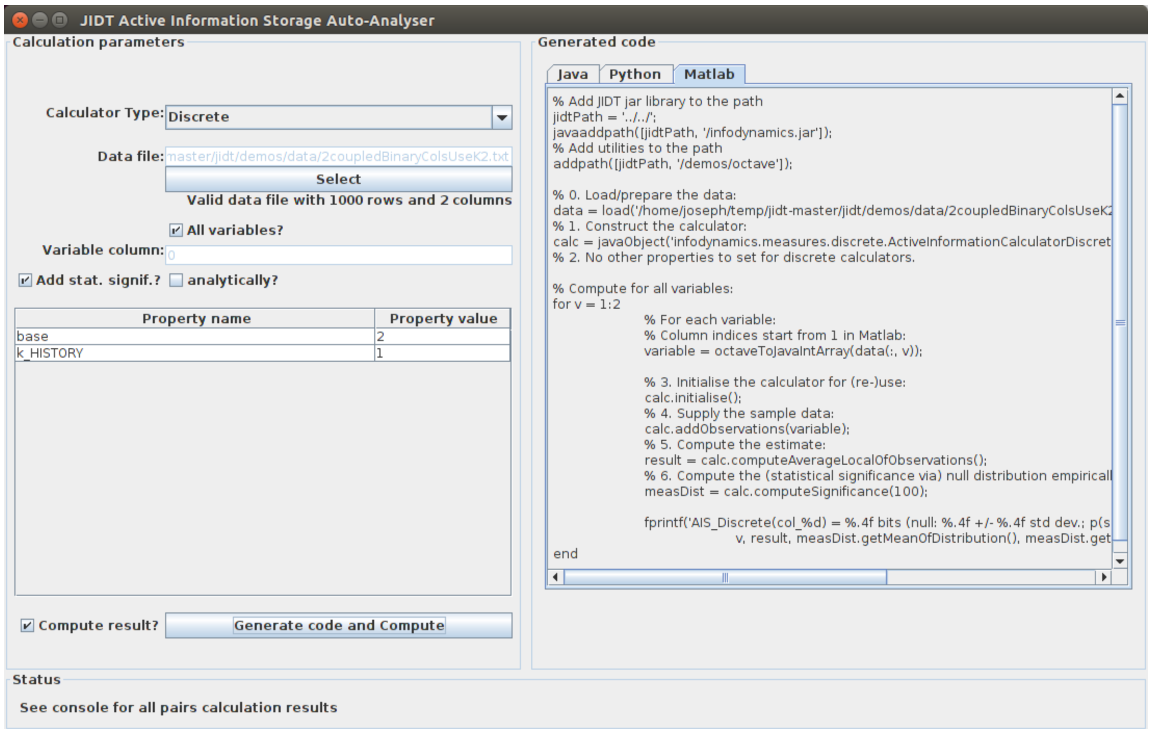
Zipser, D., Kehoe, B., Littlewort, G., and Fuster, J. (1993), "*A spiking network model of short-term active memory*", Journal of Neuroscience, 13, 8, 3406–3420

Lizier, J. T., Atay, F. M., and Jost, J. (2012), "*Information storage, loop motifs, and clustered structure in complex networks*", Phys Rev E, 86, 2, 026110

Obst, O., Boedecker, J., Schmidt, B., and Asada, M. (2013), "*On active information storage in input-driven systems*", arXiv:1303.5526

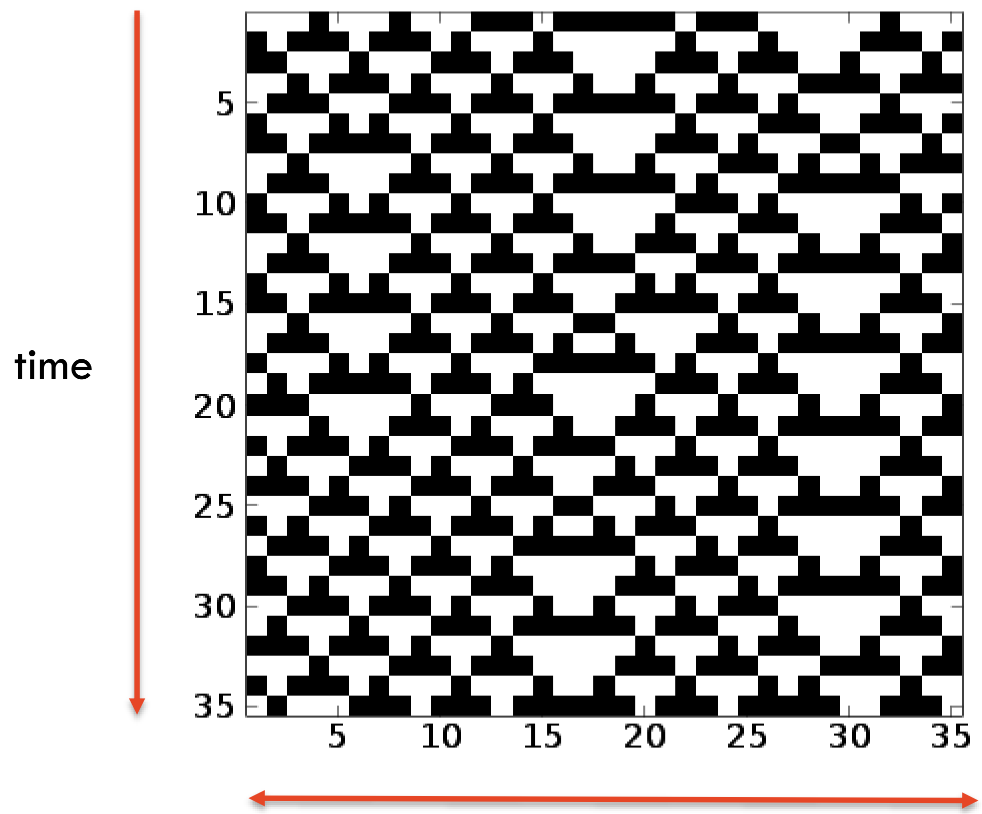
Active information storage in JIDT

- Start AIS in AutoAnalyser
- Has all types of underlying MI estimators available, same parameters as each and features (e.g. statistical significance, local values)
- Notice the important `k_HISTORY` parameter

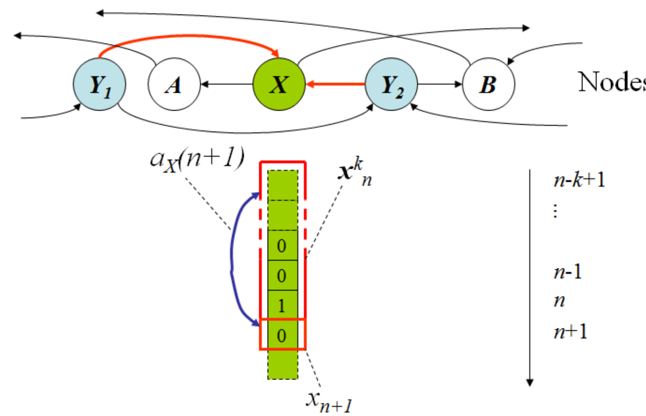


AIS: key question – how to set history length k?

– For example in Elementary Cellular Automata rule 54?



AIS: key question – how to set history length k?



- $\boldsymbol{X}_n^{(k)} = \{X_{n-k+1}, \dots, X_{n-1}, X_n\}$ is a Takens' embedding of the past state of X.
  - Can use embedding delay τ also: $\boldsymbol{X}_n^{(k,\tau)} = \{X_{n-(k-1)\tau}, \dots, X_{n-\tau}, X_n\}$
- Want to set embedding for optimal prediction of next value X_{n+1}
  - $p\left(x_{n+1} \middle| \boldsymbol{x}_n^{(k)}, x_{n-k}\right) = p(x_{n+1} | \boldsymbol{x}_n^{(k)})$
- But we have competing concerns:
  - Want k as large as possible to capture all potential memory.
  - But increasing k increases our exposure to undersampling.
- There will be a "sweet spot" in between, either:
  - Where further values from past don't actually contribute, or
  - Where there is not enough data to validate their contribution.

F. Takens, "Detecting strange attractors in turbulence", in Dynamical Systems and Turbulence, Warwick 1980, ser. Lecture Notes in Mathematics, ed. by D. Rand, L.-S. Young (Springer, Berlin, 1981), pp. 366–381

AIS: key question – how to set embedding parameters?

- Option 1: Ragwitz criteria to minimize prediction error
  - Find K nearest neighbours for each $\boldsymbol{x}_n^{(k)}$
  - Find mean of their corresponding x_{n+1}
  - Compute difference to actual x_{n+1}
  - Take mean over all points and minimize w.r.t history length k and embedding delay τ

M. Ragwitz and H. Kantz. "*Markov models from data by simple nonlinear time series predictors in delay embedding spaces*". *Phys. Rev. E*, 5(5):056201+, 2002
M. Wibral, R. Vicente, and M. Lindner. "*Transfer entropy in neuroscience*". In M. Wibral, R. Vicente, and J. T. Lizier, editors, *Directed Information Measures in Neuroscience*, pp. 3-36. Springer, Berlin/Heidelberg, 2014.

AIS: key question – how to set embedding parameters?

- Option 2: Maximize bias-corrected AIS

$$A'_X = A_X - \langle A^S_X \rangle$$

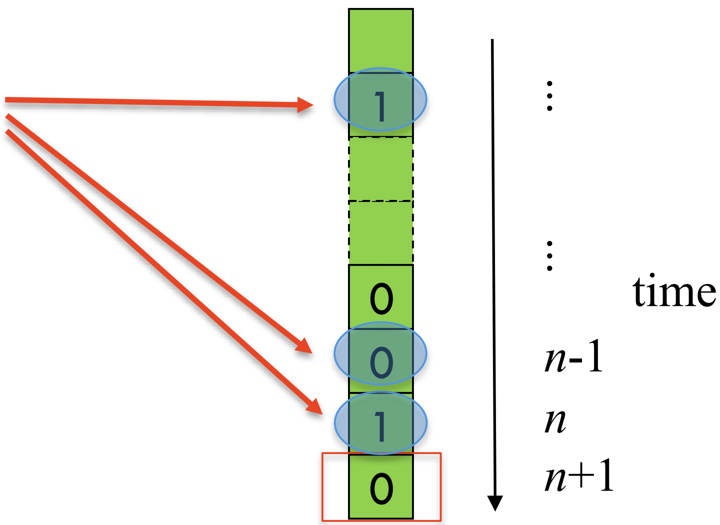
  - (where A^S_X are surrogates, created by destroying the past-next relationship)
  - w.r.t history length k and embedding delay τ

- This implies we include more points from history so long as they contribute more information about the next value than the increase in bias due to the higher dimensionality.
- For KSG estimator, bias correction is already built in!

Garland, J., James, R.G., Bradley, E. "*Leveraging information storage to select forecast-optimal parameters for delay-coordinate reconstructions*". Physical Review E 2016, 93, 022221+

AIS: key question – how to set embedding parameters?

- Option 3: Non-uniform embedding
  - Incrementally select points from past which make a statistically significant contribution beyond the points already selected.
  - Not yet implemented in JIDT but is in our higher-level IDTxl toolbox

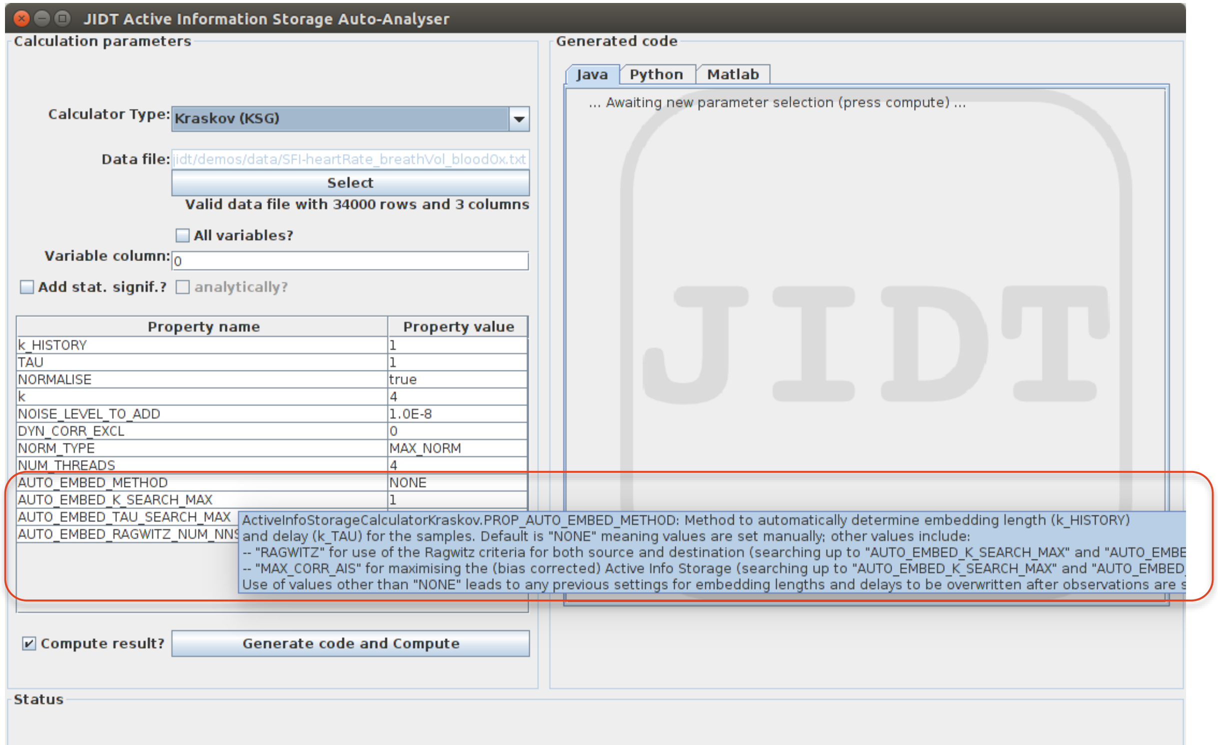


L. Faes, G. Nollo, and A. Porta, "*Information-based detection of nonlinear Granger causality in multivariate processes via a nonuniform embedding technique*", Physical Review E 83, 051112+ (2011)

P. Wollstadt, J.T. Lizier, R. Vicente, C. Finn, M. Mart/'inez-Zarzuela, P. Mediano, L. Novelli and M. Wibral, "*IDTxl: The Information Dynamics Toolkit xl: a Python package for the efficient analysis of multivariate information dynamics in networks*", arXiv:1807.10459, 2018.

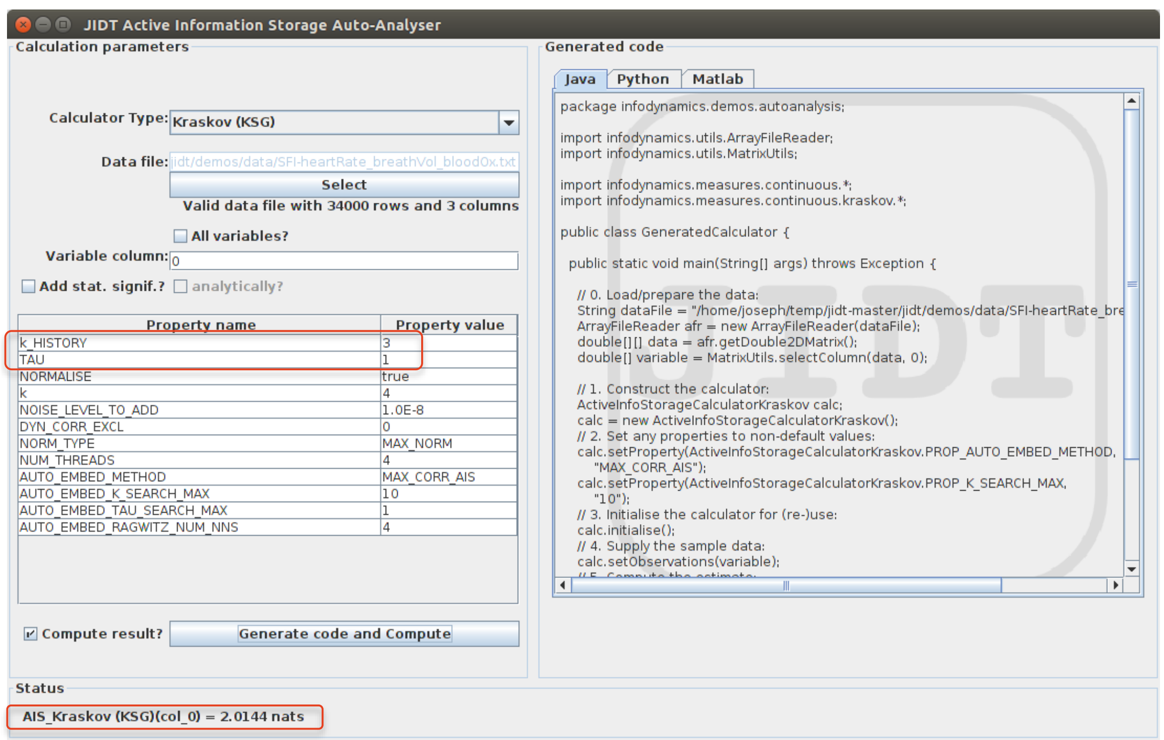
AIS: setting embedding parameters in JIDT

- Select KSG or Gaussian estimator
- Hover on `AUTO_EMBED_METHOD` property to see options:
  - `NONE`: set `k_HISTORY` and `TAU` manually.
  - `RAGWITZ`: optimal parameters to minimise prediction error scanned up to `AUTO_EMBED_K_SEARCH_MAX` and `AUTO_EMBED_TAU_SEARCH_MAX`
  - `MAX_CORR_AIS`: optimal parameters to max. bias-corrected AIS scanned up to `AUTO_EMBED_K_SEARCH_MAX` and `AUTO_EMBED_TAU_SEARCH_MAX`

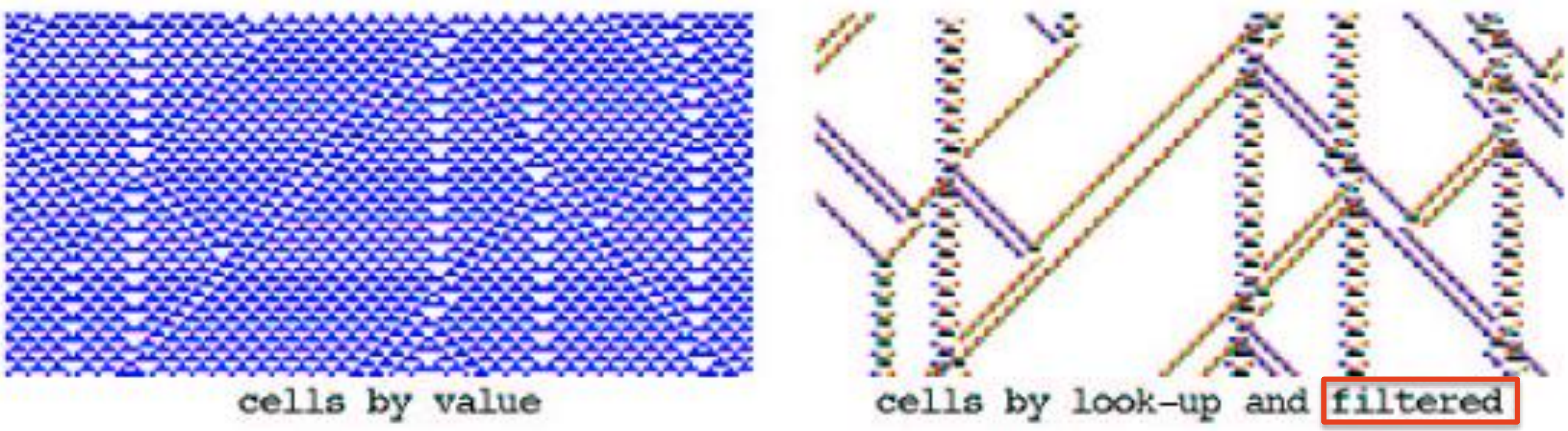


AIS: setting embedding parameters in JIDT

- Select the `SFI-heartRate_breathVol_bloodOx_extract.txt` data set
- Set `AUTO_EMBED_METHOD` to `MAX_CORR_AIS` and `AUTO_EMBED_K_SEARCH_MAX` to 10 and `AUTO_EMBED_TAU_SEARCH_MAX` to 1.
- Click Compute
- The result is returned with optimal parameters shown in `k_HISTORY` and `TAU`. You can retrieve them in code via a `getProperty()` call.



Example: Computational role of emergent structure in CAs



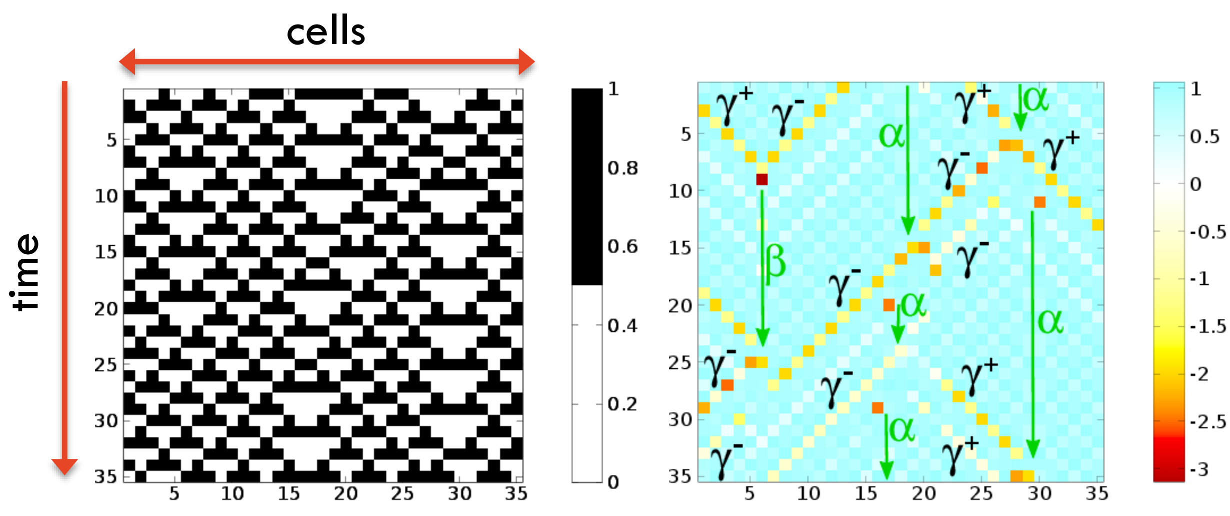
- Emergent structure:
  - Domains, blinkers
  - Particles
    - Gliders, Domain walls
  - Collisions

- Conjectured to represent
  - Information storage
  - Information transfer
    - “
  - Information modification

A. Wuensche, “Classifying cellular automata automatically: Finding gliders, filtering, and relating space-time patterns, attractor basins, and the Z parameter,” Complexity, vol. 4, no. 3, pp. 47–66, 1999. (plus image credit, © Wiley, used with permission)
C. G. Langton, “Computation at the edge of chaos: phase transitions and emergent computation,” Physica D, vol. 42, no. 1-3, pp. 12–37, 1990.
M. Mitchell, J.P. Crutchfield, R. Das, “*Evolving Cellular Automata with Genetic Algorithms: A Review of Recent Work*”, Proc. 1st Int. Conf. Evolutionary Computation and Its Applications (EvCA’96), 1996. (see p. 1/10)

Example: Computational role of emergent structure in CAs

– Go to activities to try the AIS calculator on CA data:
  – We'll compute appropriate embedding length
  – We'll compute local AIS values and see whether domains and blinkers do indeed have strong information storage values.

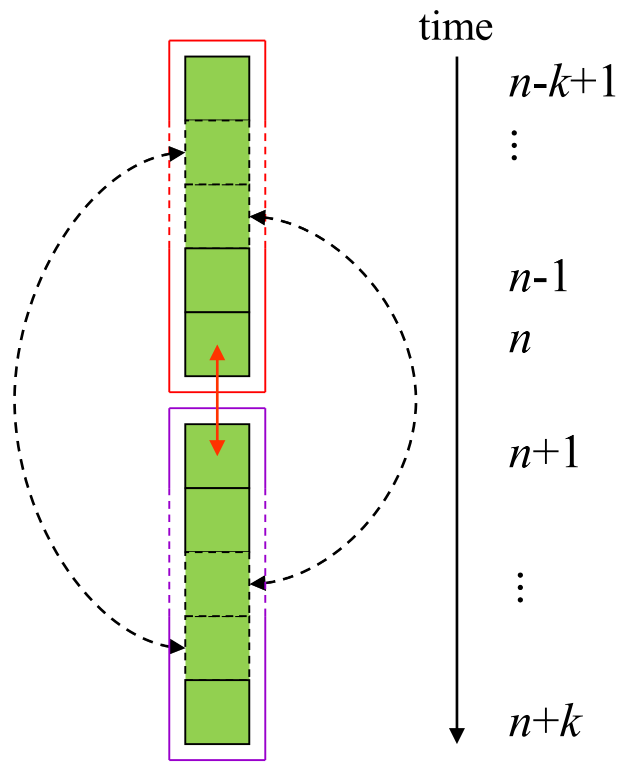


(a) Raw CA (b) Local AIS

Spoiler alert:
Blinkers and domains are dominant storage entities!

J.T. Lizier, M. Prokopenko, A.Y. Zomaya, "*Detecting Non-trivial Computation in Complex Dynamics*", Proc. ECAL, pp. 895-904 (2007).
J.T. Lizier, M. Prokopenko, & A.Y. Zomaya, "*Local measures of information storage in complex distributed computation*", Information Sciences 208, 39 (2012)
J.T. Lizier, "*JIDT: An information-theoretic toolkit for studying the dynamics of complex systems*", Frontiers in Robotics and AI, 1:11, 2014

Predictive information



– How much information about the future $\boldsymbol{X}_{n+1}^{(k+)} = \{X_{n+1}, X_{n+2}, \ldots, X_{n+k}\}$ of process X can be found in its past state $\boldsymbol{X}_{n}^{(k)} = \{X_{n-k+1}, \ldots, X_{n-1}, X_{n}\}$?

$$E_X = \lim_{k \to \infty} I(\boldsymbol{X}_{n}^{(k)}; \boldsymbol{X}_{n+1}^{(k+)})$$

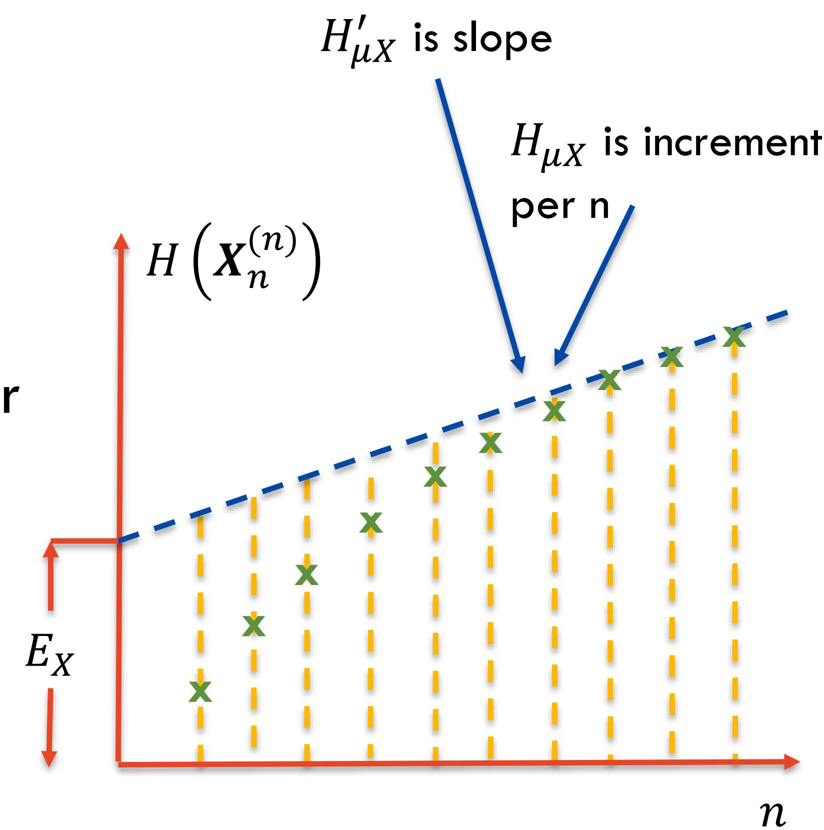
$$E_X(k) = I(\boldsymbol{X}_{n}^{(k)}; \boldsymbol{X}_{n+1}^{(k+)})$$

– Captures *all* of the information stored in the past that is used *at some point* in the future.
  – Contrast to AIS which measures the part of the stored information in use in computing the next value (so is first order component).
  – We're more interested in AIS because it focusses on the computation of the next value and is complementary to information transfer.

W. Bialek, I. Nemenman, and N. Tishby, "*Complexity through nonextensivity*", Physica A 302, 89 (2001)
J. P. Crutchfield and D. P. Feldman, "*Regularities unseen, randomness observed: Levels of entropy convergence*", Chaos 13, 25 (2003).
J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", USyd, 2010. Section 3.2.2.2

Predictive information and excess entropy

- Excess entropy quantifies total structure or memory as slowness of the approach of the conditional entropy rate estimates to their limiting value:
- Is equal to predictive information for stationary processes.



JIDT implemented

P. Grassberger, "*Toward a quantitative theory of self-generated complexity*", Int. J. Theoretical Physics 25, 907 (1986)
J. P. Crutchfield and D. P. Feldman, "*Regularities unseen, randomness observed: Levels of entropy convergence*", Chaos 13, 25 (2003).

Information storage: summary

- We’ve examined how information storage is characterised.
  - And used JIDT AutoAnalyser and extensions of code to analyse information storage in complex systems data sets.

- *Coming up*: Information transfer in complex systems

Questions