Information dynamics – Part I – Information processing in complex systems

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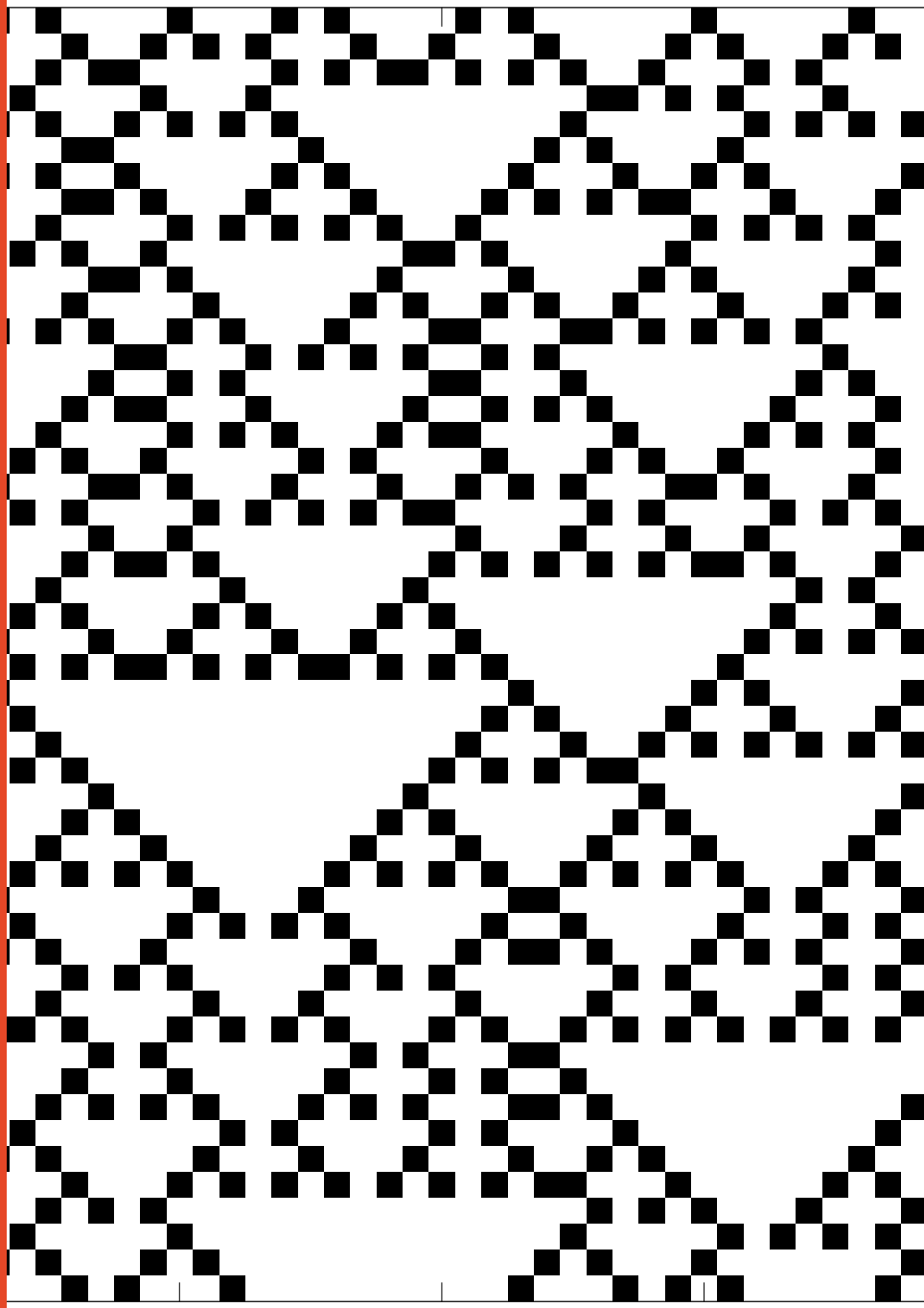

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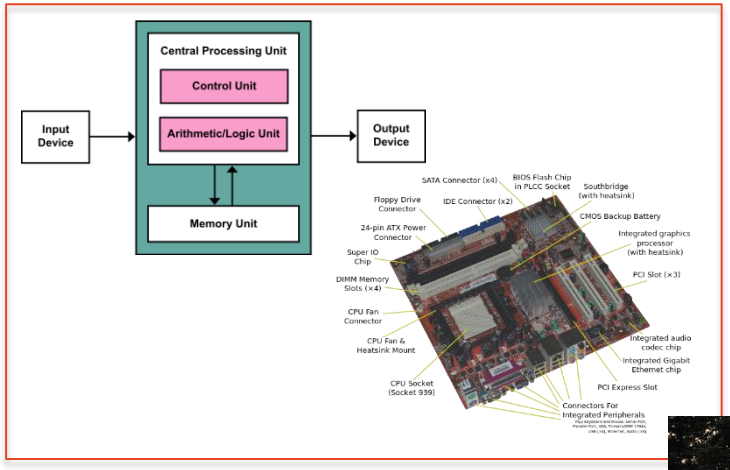
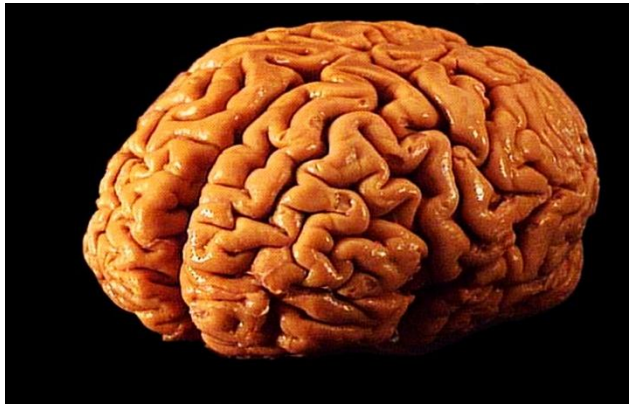
Information dynamics Part I: session outcomes

- Understand philosophy behind information dynamics approach for modelling information processing in complex systems.
- Understand entropy rate as a first consideration.

- Primary references:
  - J.T. Lizier, "JIDT: An information-theoretic toolkit for studying the dynamics of complex systems", Frontiers in Robotics and AI, 1:11, 2014; appendix A.2 and A.3

Using Turing machines, is computation easy to spot?





- For each image, consider whether the system is computing.
  - If so: What is it computing? How is it computing that: what are inputs/outputs/information? How are they manipulated?

Can we fit biological computation into dominant computer science paradigm of computation?

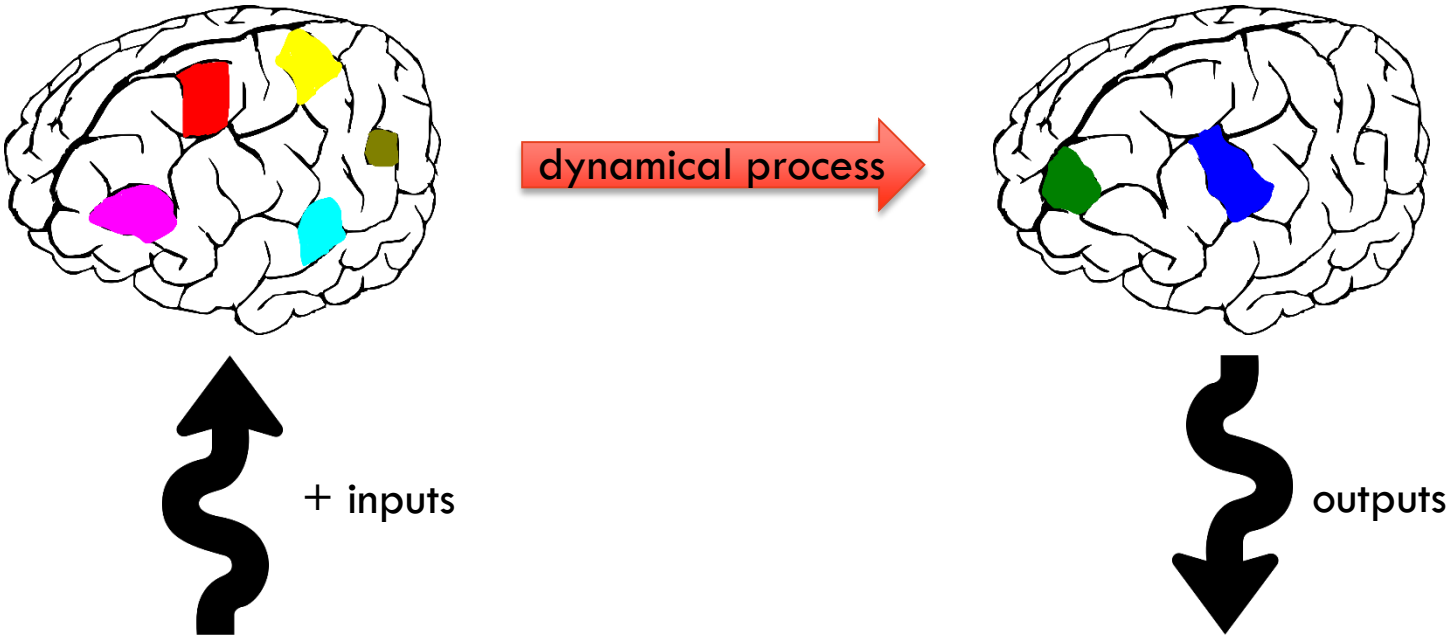
- What do you think?
- What would we look for?
- Mitchell:

	Computer science	Biological computation
What plays the role of information in the system?	Digital static tape	Analog states, patterns distributed in space and time. Gathered via statistical sampling
How is the information communicated and processed?	Deterministic, serial, error-free centralised rules	Decentralised, parallel, local, fine-grained stochastic interactions. Randomness utilised.
How does the information acquire function/purpose/meaning?	(Human) designer	Natural selection

M. Mitchell, "*Complexity: A guided tour*", New York: Oxford University Press, 2009 – chapter 12

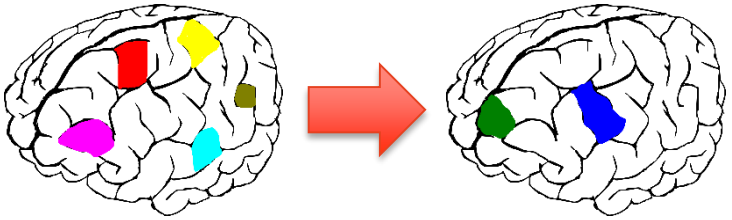
Biological computation: we need a new perspective

- Mitchell: For complex systems, the "*language of dynamical systems may be more useful than language of computation.*"



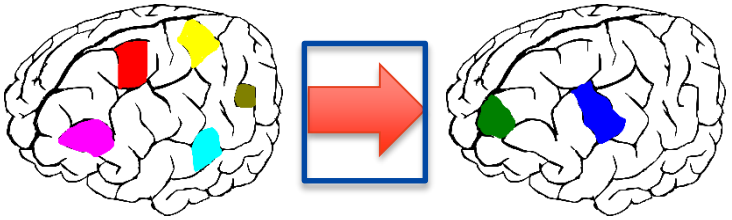
- Intrinsic information processing occurs whenever a system undergoes a dynamical process changing its initial state (+inputs) into some later state (+outputs)

Biological computation: dynamical systems perspective



- What is happening in bio- and bio-inspired information processing?
  - It's distributed, unlike a Turing machine
  - It's ongoing, unlike a Turing machine
    - Intrinsic computation, or information processing doesn't necessarily finish
  - How can we describe it in computational or informational terms?
    - Information storage, transfer and modification
    - Easy to identify (elements performing) these operations on information in a traditional PC, not so easy in biological computation

Information dynamics and computation



- We *talk* about computation as:
  - Memory
  - Signalling
  - Processing
- Distributed computation is any process that involves these features:
  - Information processing in the brain
  - Time evolution of cellular automata
  - Gene regulatory networks computing cell behaviours
  - Flocks computing their collective heading
  - Ant colonies computing the most efficient routes to food
  - The universe is computing its own future!

Information dynamics and computation

- We *talk* about computation as:
  - Memory
  - Signalling
  - Processing

- Idea: Quantify computation via:
  - Information storage
  - Information transfer
  - Information modification
- Distributed computation is any process that involves these features:
  - Information processing in the brain
  - Time evolution of cellular automata
  - Gene regulatory networks computing cell behaviours
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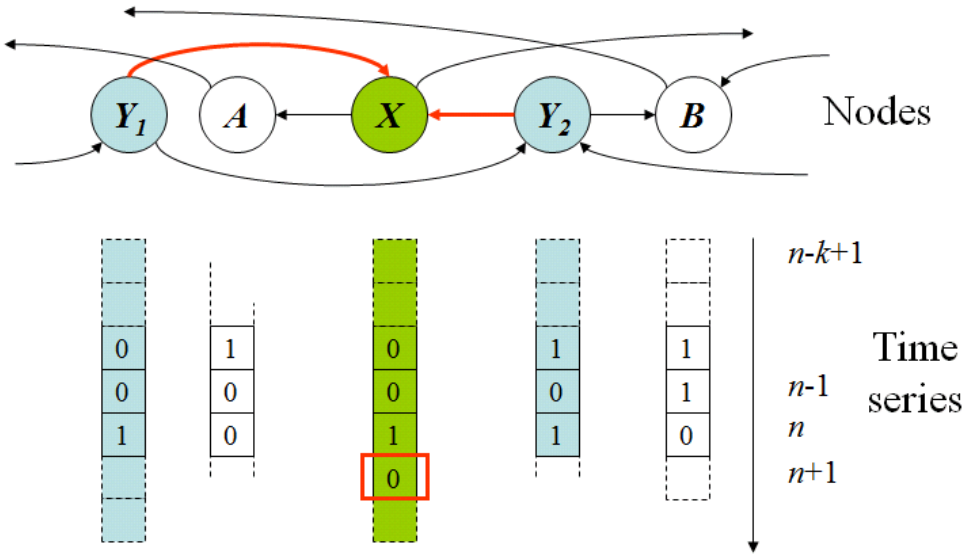
  - Ant colonies computing the most efficient routes to food
  - The universe is computing its own future!
- General idea: by quantifying intrinsic computation in the language it is normally described in, we can understand how nature computes and why it is complex.

Information dynamics

- Key question: how is the next state of a variable in a complex system computed?
- It is the output of a local computation within the system



Complex system as a multivariate time-series of states

Q: Where does the information in x_{n+1} come from, and how can we measure it?
(Where might we look?)

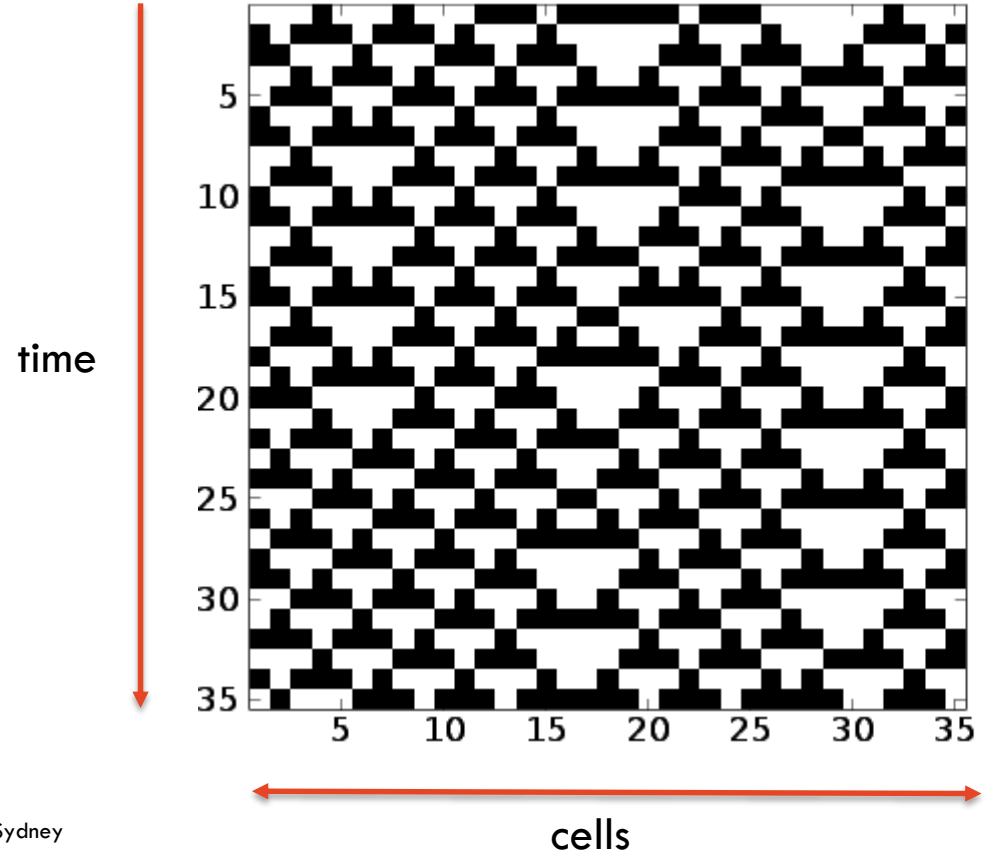
Q: Can we model the information processing in X in terms of:

- how much information was stored?
- how much was transferred, and how was this attributed?

Q: Can we partition them, do they overlap? etc.

What kinds of multivariate time series could we analyse?

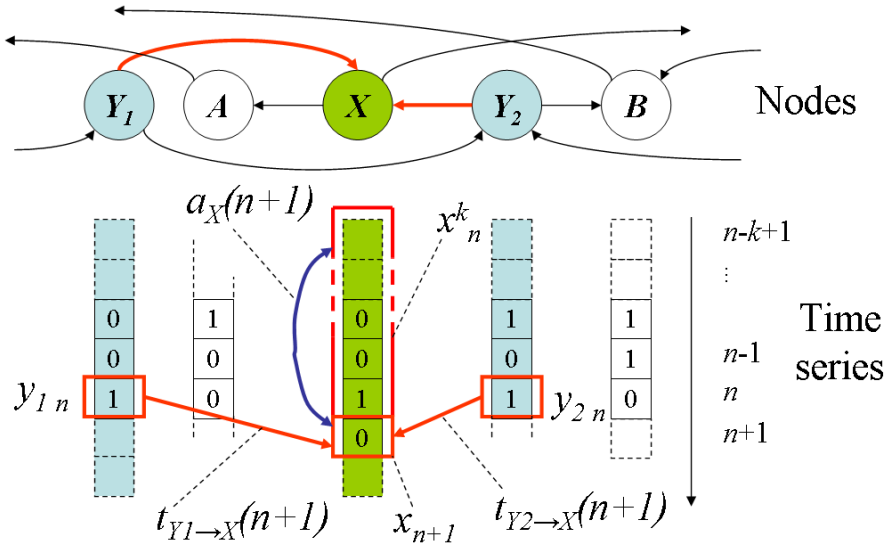
- How did we model behaviour in Scissors-Paper-Rock?
- How can we characterise the updates in cellular automata in terms of operations on information?



Elementary Cellular Automata rule 54

Information dynamics

– Studies computation of the next state of a target variable in terms of information storage, transfer and modification:



The measures examine:

- State updates of a target variable;
- Dynamics of the measures in space and time.

J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013

Notation

- We consider collections of time-series processes X, Y, Z, etc.:
  - Which each consist of random variables $\{\ldots X_{n-1}, X_n, X_{n+1}, \ldots\}$;
  - With process realisations $\{\ldots x_{n-1}, x_n, x_{n+1}, \ldots\}$;
  - For countable time indices n.

  - Denote consecutive block vector (state): $\boldsymbol{X}_n^{(k)} = \{X_{n-k+1}, \ldots, X_{n-1}, X_n\}$
  - which has realisations $\boldsymbol{x}_n^{(k)} = \{x_{n-k+1}, \ldots, x_{n-1}, x_n\}$

- Formally, we ask: "where does the information in a random variable X_{n+1} come from, in terms of other variables Y_m, Z_m, etc. for $m \leq n$?"

J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013

Entropy rate

- Historically, entropy rate was first consideration here:
  1. Measures limiting rate at which block entropies scale with block length:

$$H'_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n)$$

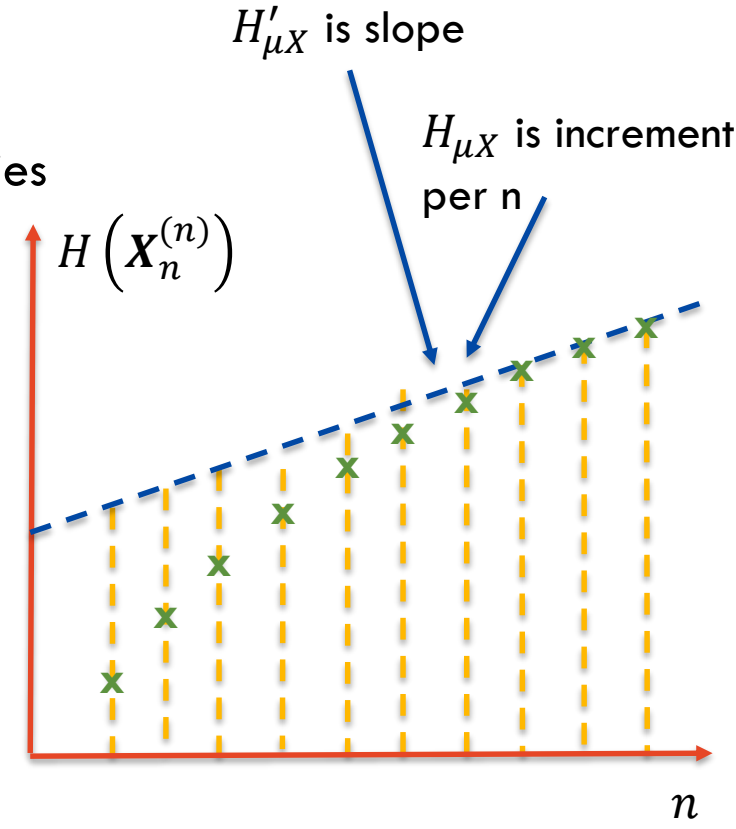
$$H'_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H\left(\boldsymbol{X}_n^{(n)}\right)$$

  2. Measures uncertainty of next R.V. given past of process:

$$H_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H(X_n | X_1, X_2, \ldots, X_{n-1})$$

$$H_{\mu X} = \lim_{n\to\infty} H\left(X_n | \boldsymbol{X}_{n-1}^{(n-1)}\right)$$

  - $H'_{\mu X} = H_{\mu X}$ for stationary processes



- Implication is that we're using past of the process as first informative source, and asking how much uncertainty remains.
- What is the coding interpretation? (Think about coding letters in English text)

T. M. Cover and J. A. Thomas. "Elements of Information Theory". Wiley-Interscience, New York, 1991. Section 4.2. *Note: primes are reversed in our notation!*

J. P. Crutchfield and D. P. Feldman, "Regularities unseen, randomness observed: Levels of entropy convergence", Chaos 13, 25 (2003).

Information dynamics Part I: summary

- We've looked at the philosophy behind the information dynamics approach for analysing information processing in complex systems.
- We've examined entropy rate as a first pass at understanding the dynamic update of time-series processes

- *Coming up*: Information storage in complex systems

Questions

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